CHAPTER 16

Credit Markets: CDS Engineering

1. Introduction

Credit derivatives have had a revolutionary effect on financial engineering. This is true in (at least) two respects. First, liquid credit derivatives permit stripping, pricing, and trading the *last* major component in financial instruments, namely the credit risk. With credit derivatives, synthetics for almost *any* instrument can be built.[1] Second, and as important, is the special role played by credit quants and financial engineers. Major broker-dealers started organizing the credit market after the development of major instruments such as options, swaps, constant maturity swaps, and swaptions was complete. New knowledge and skills were already in place. Credit markets were developed and instruments were structured using this expertise. At the end, most of the new innovations were put in place in credit markets and in structured credit. Hence, it is important to study the credit instruments, not only because they form a huge market, but also because without them many of the new financial engineering techniques would not be understood properly.

Liquid credit derivatives markets extend the creation of synthetics to assets with default risk. They also permit pricing and trading default correlation. With credit default swaps and the index products, pricing *default risk* and *default correlation* is left to the markets. In contrast, traditional approach to credit risk uses ad hoc estimates of *credit curves* and tries to model the spread dynamics.

This chapter deals with three sets of instruments. The first is the fundamental building block, credit default swaps. We will see that CDSs are a natural extension of liquid fixed-income instruments. Next, we move to index products and their tranches. Tranche trading leads to calibration of default correlation and has implications both for finance and for business cycle

[1] Without credit derivatives, creating *exact* synthetics for non-AAA-rated instruments would be possible, but it would be imperfect. A synthetic that does not use credit derivatives would require some effort in modeling credit spreads and would be ad hoc to some extent. The principle that is applied throughout this text is that pricing, hedging, and risk management should be based on liquid and tradeable securities' prices as much as possible. With credit derivatives, the ad hoc modeling aspects are minimized, and the model parameters can be calibrated to liquid markets.

analysis. Finally, as the third component of credit markets we look at various structured credit products. Structured credit is one area where new innovation takes place at a brisk rate. The chapter will also briefly review credit derivatives other than CDSs.

2. Terminology and Definitions

First, we need to define some terminology. The credit sector is relatively new in modern finance, although an ad hoc treatment of it has existed as long as banking itself. Some of the terms used in this sector come from swap markets, but others are new and specific to the credit sector. The following list is selective.

1. *Reference name.* The issuer of a debt instrument on which one is buying or selling default insurance.
2. *Reference asset.* The instrument on which credit risk is traded. Note that the credit sector adopts a somewhat more liberal definition of the basis risk. A trader may be dealing in loans but may hedge the credit risk using a bond issued by the same credit.
3. *Credit event.* Credit risk is directly or indirectly associated with some specific events (e.g., defaults or downgrades). These are important, discrete events, compared to market risk where events are relatively small and continuous.[2] The underlying credit event needs to be defined carefully in credit derivative contracts. The industry differentiates between *hard* credit events such as bankruptcy versus *soft* credit events such as restructuring.[3] We discuss this issue later in this chapter.
4. *Protection buyer, protection seller.* Protection buyer is the entity that *buys* a credit instrument such as a CDS. This entity will make periodic payments in return for compensation in the event of default. A *protection seller* is the entity that sells the CDS.
5. *Recovery value.* If default occurs, the payoff of the credit instrument will depend on the recovery value of the underlying asset at the moment of default. This value is rarely zero; some positive amount will be recoverable. Hence, the buyer needs to buy protection over and above the recoverable amount. Major rating agencies such as Moody's or Standard and Poor's have recovery rate tables for various credits which are prepared using past default data.
6. *Credit indices.* This is the most liquid part of the credit sector. A credit index is put together by first selecting a pool of reference names and then taking the arithmetic average of the CDS rates for the names included in the portfolio. There are economy-wide credit indices with investment grade and speculative grade ratings, as well as indices for particular sectors. *iTraxx* for Europe and Asia and *CDX* for the United States are the most liquid credit indices.
7. *Tranches.* Given a portfolio of reference names, it is not known at the outset which name will default, or for that matter whether there will be defaults at all. Under these conditions the structure may decide to sell, for example, the risks associated with the first 0 to 3% of the defaults. In a pool of 100 names, the risk of the first three defaults would then be transferred to another investor. The investor would receive periodic payments for bearing this risk. Similarly, the structurer may sell the risk associated with 3–6% of the defaults, etc.

[2] Wiener versus Poisson-type events provide two theoretical examples.

[3] The idea is that the default probability of a company that restructures the debt is quite different from a company that has defaulted or signaled that it will default.

The credit sector has many other sector-specific terms that we will introduce during our discussion.

2.1. Types of Credit Derivatives

Crude forms of credit derivatives have existed since the beginning of banking. These were not liquid, did not trade, and, in general, did not possess the desirable properties of modern financial instruments, like swaps, that facilitate their use in financial engineering. Banking services such as a *letter of credit*, *banker's acceptances*, and *guarantees* are precursors of modern credit instruments and can be found in the balance sheet of every bank around the world.

Broadly speaking, there are *three* major categories of credit derivatives.

1. *Credit event*–related products make payments depending on the occurrence of a mutually agreeable event. The credit default swap is the major building block here.
2. Credit *index* products that are used in trading portfolios of credit. Obviously, such indices would come with their own derivatives such as options and forwards.[4] An example would be an option written on the iTraxx Europe index.
3. The structured credit products and the index tranches.

Credit risk can be broadly grouped into two different categories: On one hand, *credit deterioration*. Widening of the underlying credit spread can indicate how credit deteriorates. On the other hand, *default risk*. This is a separate risk from credit deterioration, although it is certainly correlated with it. Default products trade default risk by separating it from credit deterioration risk.

As mentioned above, banks have issued letters of credit, guarantees, and insurance. The major distinguishing characteristic of these traditional instruments is that they transfer default risk *only*. They do not, in particular, transfer market risk or the risk of credit deterioration. Essentially, a payment is made when default occurs. With these products, no compensation changes hands when the underlying credit deteriorates. New credit *default* products share some of the properties of these old instruments. Some of the new features of credit contracts are as follows:

1. The payout is dependent on an *event* rather than an underlying price, similar to insurance products and unlike other derivatives. The dependence of a payoff on an event leads to new techniques and instruments.
2. The existence of an event leads to the issue of *recovery value*. How to determine (model) the value of an asset in case of default is now easy. Throughout this chapter we will use the assumption that the recovery rate is constant and known at a level R.
3. The process of *settling* credit contracts is more complex than in other markets. In the case of *physical delivery* of the underlying, this does not present a major problem. The protection seller will be the legal owner of the defaulted instrument and may take necessary legal steps for recovery. But if the contract is cash-settled, then neither party has legal recourse to the borrower unless the party owns the underlying credit directly. For this reason, the industry prefers physical delivery, and a large majority of default swaps settle this way.

[4] Forwards on the indices are not traded. Eurex tried to launch a contract on the indices, but this was not used by the major banks in Europe.

We will address the additional characteristics of default products when we study credit default swaps in more detail. In the next section, we will look at the most liquid credit derivatives in more detail, and study the financial engineering of credit default swaps.

3. Credit Default Swaps

The major building block of the credit sector is the credit default swap, introduced in the first chapter as an example in the swap family. It is, however, a major category. A typical default swap from the point of view of a protection *seller* is shown in Figure 16-1. The CDS seller of a particular credit denoted by i receives a preset coupon called the CDS rate. The CDS expires at time T. The CDS spread is denoted by $c^j_{t_0}$ and is set at time t_0. A payment of $c^j_{t_0}\delta N$ is made at every t_i. The j represents the reference name. If no default occurs until T, the contract expires without any other payments. On the other hand, if the name i defaults during $[t_0, T]$, the CDS seller has to compensate the counterparty by the insured amount, N dollars. Against this payment of cash, the protection buyer has to deliver eligible debt instruments with *par value* N dollars. These instruments will be from a *deliverable basket*, and are clearly specified in the contract at time t_0. Obviously, one of these instruments will, in general, be cheapest-to-deliver in the case of default, and all players may want to deliver that particular underlying.

Later in this chapter we will consider additional properties of the default swap market that a financial engineer should be aware of. At this point, we discuss the engineering aspects of this product. This is especially important because we will show that a default swap will fall naturally as the residual from the decomposition of a typical risky bond. In fact, we will take a risky bond and decompose it into its components. The key component will be the default swap. This natural function played by default swaps partly explains their appeal and their position as the leading credit instrument.

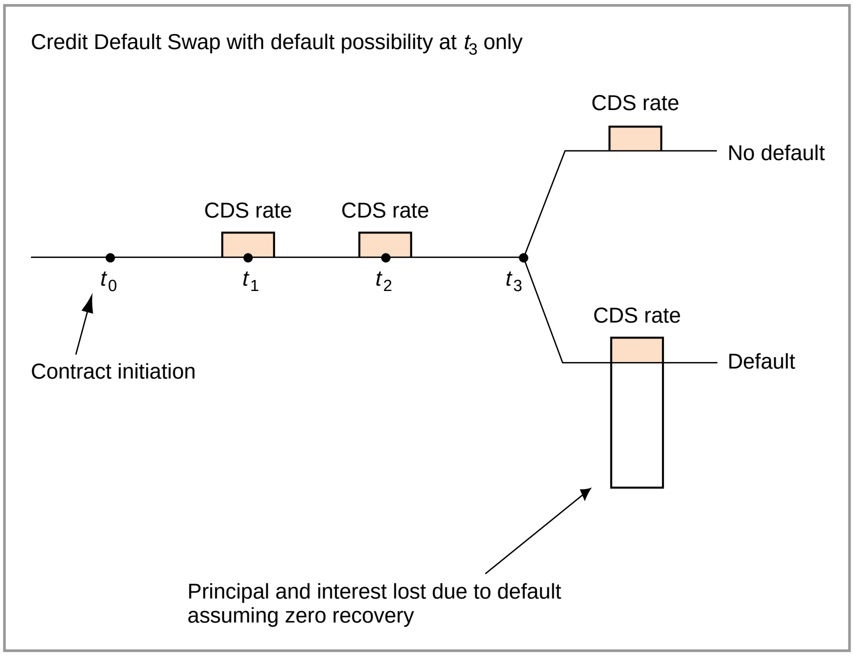

FIGURE 16-1

We discuss the creation of a default swap by using a specific example. The example deals with a special case, but illustrates almost all the major aspects of engineering credit risk. Many current practices involving synthetic collateralized debt obligations (CDOs), credit linked notes (CLNs), and other popular credit instruments can be traced to the discussion provided next.

Independently, this section can be seen as another example of engineering cash flows. We show how the static replication methods change when default risk is introduced into the picture. Essentially, the same techniques are used. But the creation of a satisfactory synthetic becomes possible only if we add CDSs to other standard instruments.

3.1. Creating a CDS

The steps we intend to take can be summarized as follows. We take a bond issued by the reference name j that has *default risk* and then show how the cash flows of this bond can be decomposed into simpler, more liquid constituents. Essentially we decompose the bond risk into two—one depending on market volatility only, the other depending on the reference home's likelihood of default. Credit default swaps result naturally from this decomposition.

Our discussion leads to a new type of *contractual equation* that will incorporate credit risk. We then use this contractual equation to show how a credit default swap can be created, hedged, and priced in theory. The contractual equation also illustrates some of the inherent difficulties of the hedging and pricing process in practice. At the end of the section, we discuss some practical hedging and pricing issues.

3.2. Decomposing a Risky Bond

We keep the example simple in order to illustrate the fundamental issues more clearly. Consider a "risky" bond, purchased at time t_0, subject to default risk. The bond does not contain any implicit call and put options and pays a coupon of Co_{t_0} annually over three years. The bond is originally sold at par.[5]

We make two further *simplifying assumptions* which can be relaxed with little additional effort. These assumptions do not change the essence of the engineering, but significantly facilitate the understanding of the credit instrument. First, we assume that, in the case of default, the recovery value equals the *known* constant R. Second, and without much loss of generality, we assume that the default occurs *only* at settlement dates t_i. Finally, to keep the graphs tractable we assume that settlement dates are annual, and that the maturity of the bond is $T = 3$ years.

Figure 16-2 shows the cash flows implied by this bond. The bond is initially purchased for 100, three coupon payments are made, and the principal of 100 is returned *if* there is no default. On the other hand, if there is default, the bond pays nothing. The dependence on default is shown with the fork at times t_i. At each settlement date there are two possibilities and the claim is *contingent* on these.

How do we reverse engineer these cash flows and convert them into liquid financial instruments? We answer this question in steps.

First, we need to introduce a useful trick that will facilitate the application of static decomposition methods to defaultable instruments. We do this in Figure 16-3. Remember that our goal is to *isolate* the underlying default risk using a *single* instrument. This task will be greatly

[5] The latter is an assumption made for convenience and is rarely satisfied in reality. Bonds sold at a discount or premium need significant adjustments in their engineering as discussed below. However, these are mostly technical in nature.

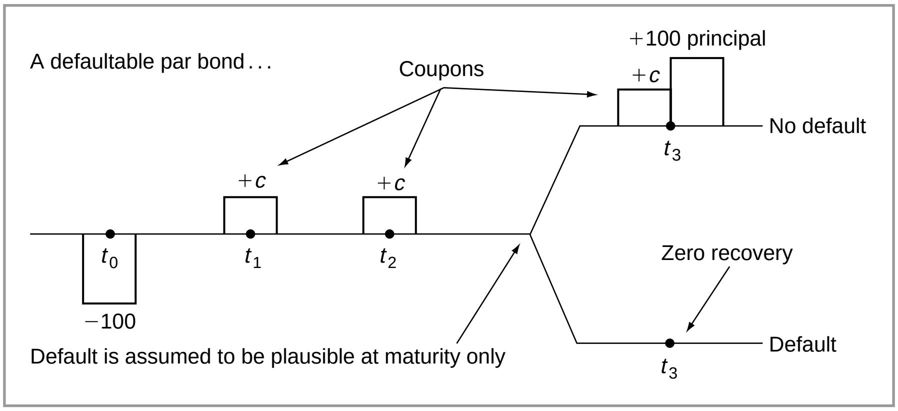

FIGURE 16-2

simplified if we add *and* subtract the amount $(1 + Co_{t_0})N$ to the cash flows in the case of default at times t_i. Note that this does not change the original cash flows. Yet, it is useful for isolating the inherent credit default swap, as we will see.

Now we can discuss the decomposition of the defaultable bond. The bond in Figure 16-2 contains three different types of cash flows:

1. Three coupon payments on dates t_1, t_2, and t_3. We strip these fixed cash flows and place them in Figure 16-3b. Although the coupons are risky, we can still extract three default-free coupon payments from the bond cash flows due to the trick used. To get the default-free coupon payments, we simply pick the positive (Co_{t_0}) 100 at the default state for times t_i of Figure 16-3a. Note that this leaves the negative (Co_{t_0}) 100 in place.
2. Initial and final payment of 100 as shown in Figure 16-3c. Again, adding and subtracting 100 is used to obtain a default-free cash flow of 100 at time t_3. These two cash flows are then carried to Figure 16-3c. As a result, the negative payment of 100 in the default state of times t_i remains in Figure 16-3a.
3. All remaining cash flows are shown in Figure 16-3d. These consist of the negative cash flow $(1 + Co_{t_0})100$ that occurs in the time t_3 default state. This is detached and placed in Figure 16-3d.

The next step is to convert the three cash flow diagrams in Figures 16-3b, 16-3c, and 16-3d into recognizable and, preferably, liquid contracts traded in the markets. Remember that to do this, we need to add and subtract arbitrary cash flows to those in Figures 16-3b, 16-3c, and 16-3d while ensuring that the following three conditions are met:

- For each cash flow added, we have to *subtract* the same amount (or its present value) at the same t_i from one of the Figures 16-3b, 16-3c, or 16-3d.
- These new cash flows should be introduced to make the resulting instruments as liquid as possible.
- When added back together, the modified Figures 16-3b, 16-3c, and 16-3d should give back the original bond cash flows in Figure 16-3a. This way, we should be able to recover the cash flows of the defaultable bond.

This process is displayed in Figure 16-4. The easiest cash flows to convert into a recognizable instrument are those in Figure 16-3b. If we add floating Libor-based payments, L_{t_i} at times t_1, t_2, and t_3, these cash flows will *look like* a fixed-receiver interest rate swap. This is good because

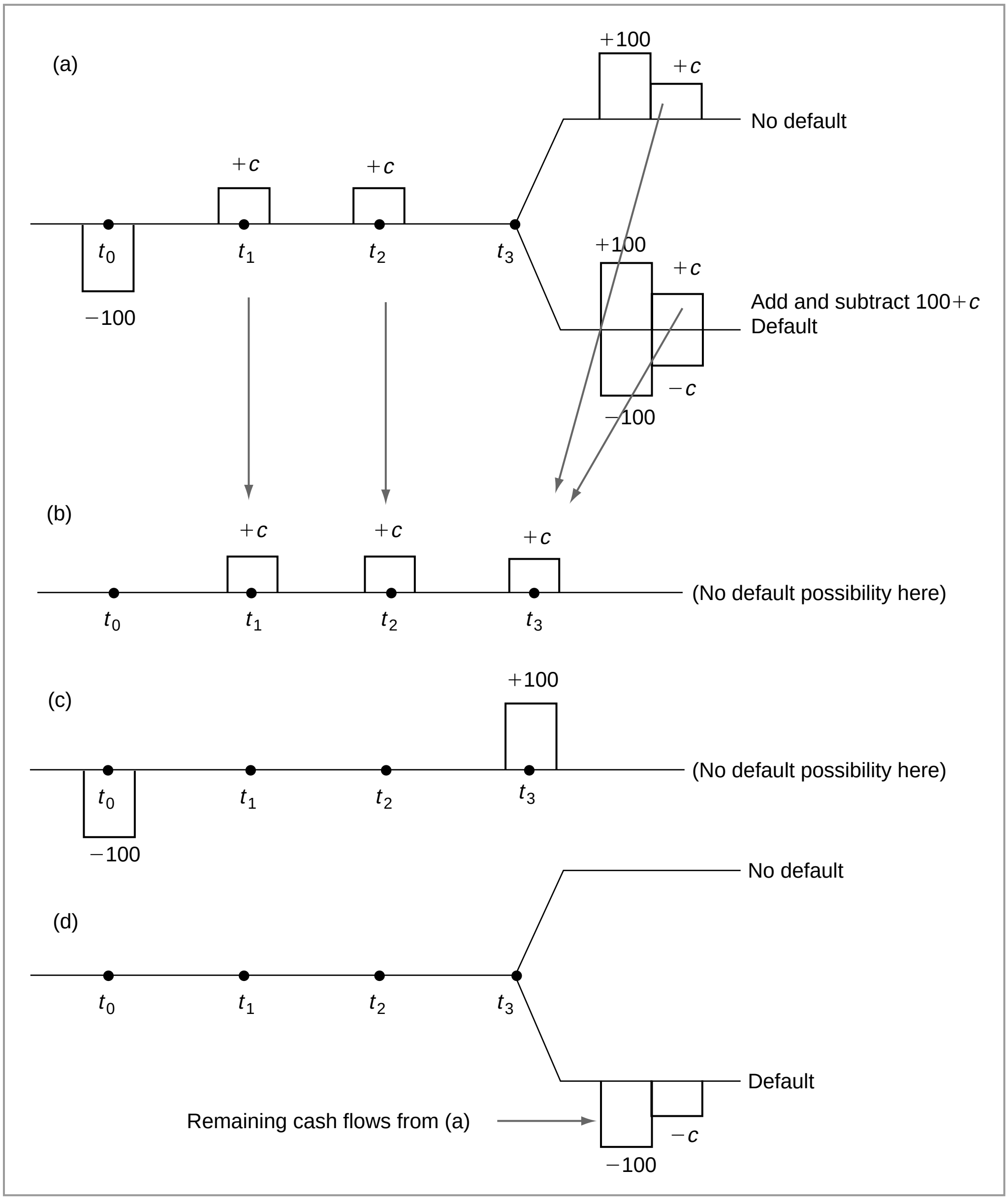

FIGURE 16-3

swaps are very liquid instruments. However, one additional modification is required. The fixed-receiver swap rate, s_{t_0}, is less than the coupon of a par bond issued at time t_0, since the bond can default while the swap is subject only to a counterparty risk. Thus, we have

$$s_{t_0} \leq Co_{t_0} \tag{1}$$

The difference, denoted by c_{t_0},

$$c_{t_0} = Co_{t_0} - s_{t_0} \tag{2}$$

is the *credit spread over the swap rate*. This is how much a credit has to pay over and above the swap rate due to the default possibility. Note that we are defining the credit spread as a

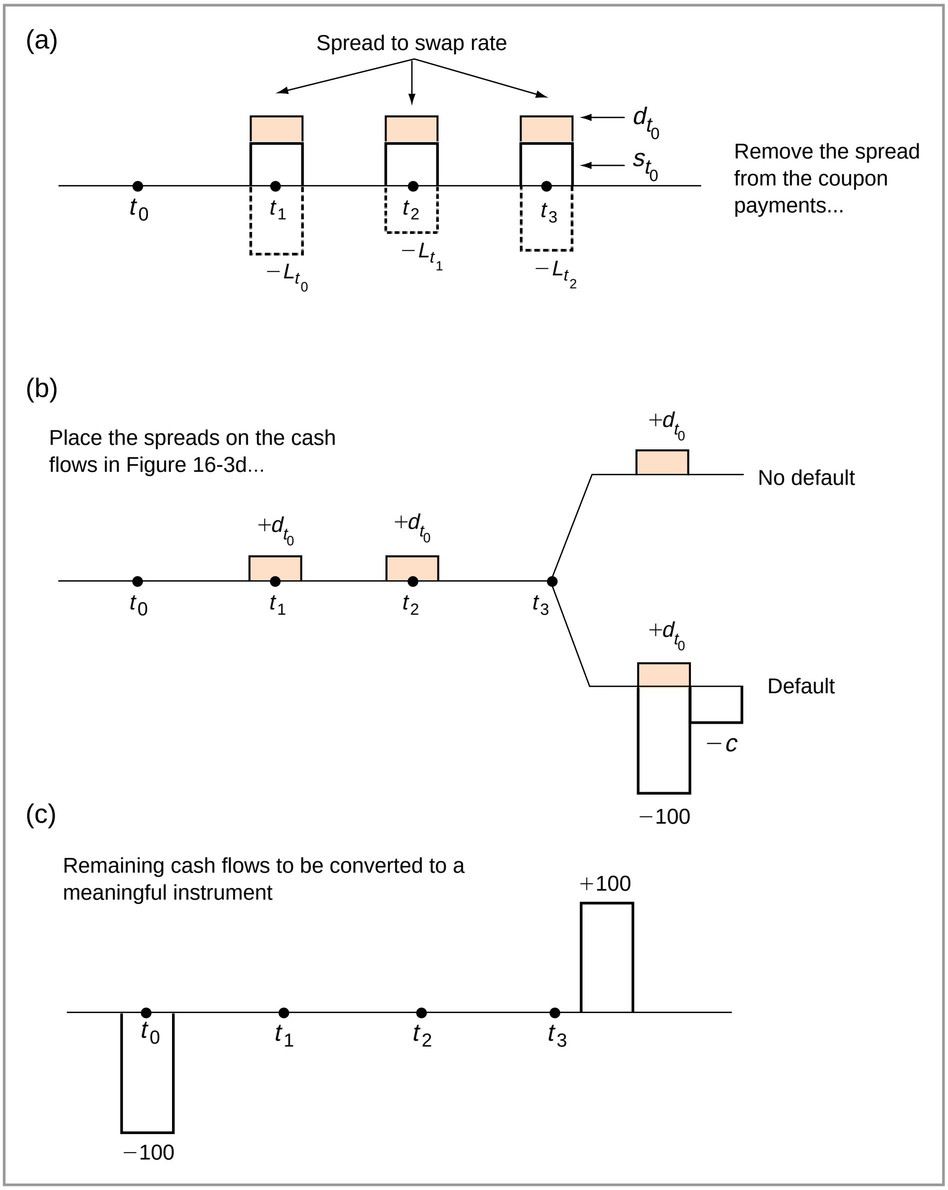

FIGURE 16-4

spread over the corresponding swap rate and *not* over that of the treasuries. This definition falls naturally from cash flow decompositions.[6]

Thus, in order for the cash flows in Figure 16-4a to be equivalent to a receiver swap, we need to subtract c_{t_0} from each coupon as done in Figure 16-4a. This will make the fixed receipts equal the swap rate:

$$Co_{t_0} - c_{t_0} = s_{t_0} \tag{3}$$

The resulting cash flows become a true *interest rate swap*.

[6] We should also mention that AAA credits have sub Libor funding cost.

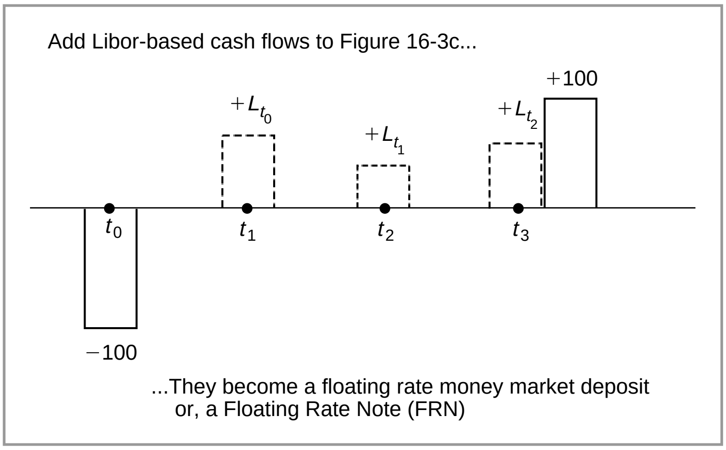

FIGURE 16-5

Next question is where to *place* the counterparts of the cash flows c_{t_0} and L_{t_i} that we just introduced in Figure 16-4a. After all, unless the same cash flows are placed *somewhere* else with opposite signs, they will not cancel out, and the resulting synthetic will not reduce to a risky bond.

A natural place to put the Libor-based cash flows is shown in Figure 16-5. Nicely, the addition of the Libor-related cash flows converts the figure into a *default-free money market deposit* with tenor δ. This deposit will be rolled over at the going floating Libor rate. Note that this is also a liquid instrument.[7]

The final adjustment is how to compensate the reduction of Co_{t_0}'s by the credit spread c_{t_0}. Since the first two instruments are complete, there is only one place to put the compensating c_{t_0}'s. We add the c_{t_0} to the cash flows shown in Figure 16-3d, and the result is shown in Figure 16-4b. This is the critical step, since we now have obtained a *new* instrument that has fallen naturally from the decomposition of the risky bond. Essentially, this instrument has potentially three receipts of c_{t_0} dollars at times t_1, t_2, and t_3. But if *default* occurs, the instrument will make a compensating payment of $(1 + Co_{t_0})100$ dollars.[8]

To make sure that the decomposition is correct, we add Figures 16-4a, 16-4b, and 16-5 vertically and see if the original cash flows are recovered. The vertical sum of cash flows in Figures 16-4a, 16-4b, and 16-5 indeed replicates exactly the cash flows of the defaultable bond.

The instrument we have in Figure 16-4b is equivalent to *selling* protection against the default risk of the bond. The contract involves collecting *fees* equal to c_{t_0} at each t_i until the default occurs. Then the protection buyer is compensated for the loss. On the other hand, if there is no default, the fees are collected until the expiration of the contract and no payment is made. We call this instrument a *credit default swap* (CDS).

3.3. A Synthetic

The preceding discussion shows that a defaultable bond can be decomposed into a portfolio made up of (1) a fixed receiver interest rate swap, (2) a default-free money market deposit,

[7] Alternatively, we can call it a floating rate note (FRN).

[8] According to this, in the case of default, the total *net* payment becomes $(1 + Co_{t_0})100 - c_{t_0}100$.

and (3) a credit default swap. The use of these instruments implies the following contractual equation:

Defaultable bond on the credit	=	Receiver swap	+	Default-free deposit	+	CDS on the credit		(4)

By manipulating the elements of this equation using the standard rules of algebra, we can obtain synthetics for every instrument in the equation. In the next section, we show two applications.

3.4. *Using the Contractual Equation*

As a first application, we show how to obtain a *hedge* for a long or short CDS position by manipulating the contractual equation. Second, we discuss the implied pricing and the resulting real-world difficulties.

3.4.1. Creating a Synthetic CDS

First, we consider the way a CDS would be hedged. Suppose a market maker sells a CDS on a certain name. How would the market maker hedge this position while it is still on his or her books?

To obtain a hedge for the CDS, all we need to do is to manipulate the contractual equation obtained above. Rearranging, we obtain

Defaultable bond issued by the credit	−	Receiver swap	−	Default-free deposit	=	CDS on the credit		(5)

Remembering that a negative sign implies the opposite position in the relevant instrument, we can write the formal synthetic for the credit default swap as

Sell CDS on the credit	=	Short a risky bond on credit	+	Payer swap	+	Make a default-free loan		(6)

The market maker who sold such a CDS and provided protection needs to take the *opposite* position on the right-hand side of this equation. That is to say, the credit derivatives dealer will first *short* the risky bond, deposit the received 100 in a default-free deposit account, and contract a receiver swap. This and the long CDS position will then "cancel" out. The market maker will make money on the bid-ask spread.

3.4.2. Negative Basis Trades

The second application of the contractual equation is referred to as *Negative Basis Trades*. Negative basis trades are an important position frequently taken by the traders in credit markets. A discussion of the trade provides a good example of how the contractual equation defining the CDS contracts changes in the real world.

The contractual equation that leads to the creation of a credit default swap can be used to construct a synthetic CDS that can be used against the actual one. Essentially, with a negative basis trade one would buy a bond that pays the par-yield Co_{t_0} while at the same time, buy insurance on the same bond. Clearly such a position has no default risk, and makes sense if the coupon minus the swap rate is greater than the CDS premium payments

$$s_{t_0} + c_{t_0} < Co_{t_0} \tag{7}$$

This is in fact the case of *negative basis*.

Normally, the insurance on a default risky bond should be "slightly" higher than the credit risk spread one would obtain from the bond. This *basis* is in general positive. Otherwise, if the bond spread is larger, then one would buy the bond *and* buy insurance on it. This would be a perfect arbitrage. This is exactly what a *negative basis* is. The reading below suggests when and how this may occur.

EXAMPLE:

A surge of corporate bond and structured finance issuance this past month has pushed risk premiums on credit default swaps down and those on investment-grade corporate bonds up, nearly erasing the difference between the two asset-classes. If this trend continues, it has the potential to create new trading opportunities for investors who can take positions in both bonds and derivatives. Analysts are expecting to see more so-called "negative basis trades" as a result.

January has been a particularly active month in the US primary corporate bond market, with over $60 billion issued in investment-grade debt alone. This supply helps to widen risk premiums on corporate bonds. At the same time, there has been no shortage of synthetic collateralized debt obligations (CDOs), which has helped narrow CDS spreads. A corporate credit default swap contract features a seller of protection and a buyer of protection. The seller is effectively long that company's debt while the buyer is short. When a synthetic CDO is created, dealers sell a certain amount of credit protection, which helps compress CDS risk premiums.

Typically, credit default swap risk premiums trade at wider levels than comparable bond risk premiums. This is partly because it is easier to take a short position via a CDS rather than a bond. Another reason that CDS risk premiums trade wider is the cheapest-to-deliver option. In the event of a bankruptcy, the seller of protection in a CDS contract will make a cash payment to the buyer in exchange for the bonds of the bankrupt company. However, the seller of protection will receive the cheapest-to-deliver bond from the protection buyer.

Once the basis between a CDS risk premium and cash bond reverses and becomes negative, it can become advantageous to buy the bond and buy protection through a

> *credit default swap. In such a trade, known as a negative basis trade, the investor has hedged out their credit risk, but is earning more on their bond position than they are paying out on their CDS position. (IFR, January 2006).*

Negative basis trades become a possibility due to leveraged buyout (LBO) activity as well. During an LBO the buyer of the company issues large amounts of debt, which increases the debt to equity ratio on the balance sheet. Often, rating agencies downgrade such LBO targets several notches which makes LBO candidates risky for the original bond holder. Bond holders, hearing that a company is becoming an LBO target, may sell before the likely LBO; bond prices may drop suddenly and the yields may spike. On the other hand the CDS rates may move less since LBO is not necessarily going to increase the *default* probability. As a result, the basis may momentarily turn negative.

3.5. Measuring Credit Risk of Cash Bonds

Many credit and fixed income strategies involve arbitraging between cash and synthetic instruments. CDS is the synthetic way of taking an exposure to the default of a single name credit. It is a clean way of trading default. But, cash bonds contain default risk as well as interest rate and curve risk. How would we strip from a defaultable bond yield the component that is being paid due to default risk? In other words, how do we obtain the equivalent of the CDS rate from a defaultable bond *in reality?*

This question needs to be answered if we are to take arbitrage positions between cash and derivatives; for example, when we have to make a decision whether cash bonds are *too expensive* or not relative to the CDS of the same name.

At the outset the question seems unnecessary, since we just developed a contractual equation for the CDS. We showed that if we combined the cash bond and a vanilla interest rate swap accordingly, then we would obtain a synthetic FRN that paid Libor plus a spread. The spread would equal the CDS rate. In other words, take the par yield of a par bond, subtract from this the comparable interest rate swap rate, and get the equivalent of the CDS rate included in the bonds yield.

This is indeed true, except for one major problem. The formula is a good approximation only if all simplifying assumptions are satisfied and if the cash bonds are selling *at par*. It is only then that we can straightforwardly put together an *asset swap* and strip the credit spread.[9] If the bond is not selling for par, we would need further adjustments.

There are two major methods used to obtain a measure of the default risk contained in a bond that does not trade at par. The first is to calculate the *asset swap spread* and the second is to calculate the so-called *Z-spread*. We discuss these two practical concepts next and give examples.

3.5.1. Asset Swap

Asset swap spread is one way of calculating the credit spread associated with a default risky bond. Essentially it converts the risky yield into a *Libor plus credit spread*, using an interest rate swap (IRS).

In order to create a position equivalent to selling protection, we must buy the defaultable bond and get in a payer interest rate swap. Note that the IRS will be a par swap here, while the

[9] There are also some day-count adjustments that we assumed away.

bond might be selling for a discount or premium.[10] So adjustments are needed in reality since the bond sells either at a discount or premium.[11]

In the asset swap, the bond cash flows are discounted using the corresponding zero coupon swap rates. A spread (called the asset swap spread) is added to (subtracted from) the bond cash flows so that the resulting bond price equals the market price.

The formula for calculating the asset swap spread is as follows: First use the *zero-coupon* swap curve to calculate the discount factors. By definition, *zero-coupon swap curve* discounts are obtained using the corresponding forward rates f_{t_j}, and the equation,

$$(1 + s_{t_0}^i \delta)^i = \prod_{j=0}^{i-1} (1 + f_{t_j} \delta) \tag{8}$$

Then calculate the discount factors

$$B(t_0, t_i) = \frac{1}{(1 + s_{t_0}^i \delta)^i} \tag{9}$$

Once this is done, form the *annuity factor*, A.

$$A = \sum_{i=1}^{n} B(t_0, t_i)\delta \tag{10}$$

Next, calculate the value of the bond cash flows using these factors:

$$\tilde{P}_{t_0} = \sum_{i=1}^{n} \frac{Co_{t_i}}{(1 + s_{t_0}^i \delta)^i} \tag{11}$$

The asset swap spread is the $\tilde{a}_{t_0}$ that solves the equation:

$$\tilde{P}_{t_0} - P_{t_0} = \tilde{a}_{t_0} \sum_{i=1}^{n} B(t_0, t_i)\delta \tag{12}$$

Thus, the $\tilde{a}_{t_0}$ is how much one needs to be paid extra in order to compensate for the difference between the market price and the theoretical price implied by the default-free swap curve. This is the case, since if there were no default risk the value of the bond would be $\tilde{P}_{t_0}$ which is greater than the market price P_{t_0}. The $\tilde{a}_{t_0}$ is a measure that converts this price differential into an additional spread.

We can show how asset swaps are structured using this last equation. A par bond paying the swap rate s_{t_0} satisfies the equation:

$$100 = \sum_{i=1}^{n} B(t_0, t_i) s_{t_0} \delta + B(t_0, t_n) 100 \tag{13}$$

[10] Remember how swaps were engineered: We added an FRN position to a par bond paying the swap rate s_{t_0}. This leads to the par swaps traded in the markets.

[11] In an asset swap the IRS and the bond position are written on separate tickets. If there is default, the swap position needs to be closed separately and this may involve an extra exposure. The swap position may be making or losing money when default occurs.

Thus after adding 100 to both sides the previous equation can be written as:

$$\tilde{P}_{t_0} - (P_{t_0} - 100) = (s_{t_0} + \tilde{a}_{t_0}) \sum_{i=1}^{n} B(t_0, t_i)\delta + B(t_0, t_n)100 \tag{14}$$

where the second term on the left-hand side is the upfront payment (receipt) upf_{t_0}

$$upf_{t_0} = (P_{t_0} - 100) \tag{15}$$

The latter is negative if the bond is selling at a discount and positive if the bond is at a premium.

We interpret this equation as follows. Suppose the bond is trading at a discount, then an upfront payment of upf_{t_0} plus an IRS contracted at a rate $s_{t_0} + \tilde{a}_{t_0}$ is equivalent to the present value of the coupon payments of the bond. In other words, a par swap rate has to be *augmented* by $\tilde{a}_{t_0}$ in order to compensate for the bond's default risk. Note that during this exercise we worked with *risk-free discount factors*. This will change with the next notion.

3.5.2. The Z-Spread

The so-called Z-spread is another way of calculating the credit spread. It gives a result similar to the asset swap spread but is not necessarily the same.

In order to calculate the Z-spread, the cash flows generated by a default risk bond will be discounted by the *zero-coupon swap rate* s_{t_0} augmented by a spread z_{t_0}, so that the sum equals the market price of the bond. That is to say we have

$$P_{t_0} = \sum_{i=1}^{n} \frac{cf_i}{(1 + (s_{t_0}^i + z_{t_0})\delta)^i} \tag{16}$$

We solve this equation for the unknown z_{t_0}. Here cf_{t_i} are the cash flows received at time t_i and are made of coupon payments Co_{t_i} and possibly of the principal.

According to this, the zero-coupon swap curve is adjusted in a parallel fashion so that the present value of the cash flows equals the bond price. The Z-spread is the amount of parallel movement in the zero-coupon swap curve needed to do this.

Note that during the calculation of the Z-spread we worked with a measure of *risky discount factors*.[12] The major difference between the Z-spread and the asset swap spread arises from the discount rates used. Asset swaps use zero-coupon swap rates whereas Z-spread uses zero-coupon swap rates *plus* the Z-spread.

In this sense the Z-spread method uses a stream of risky discounts from the *whole risky curve* to adjust the future cash flows of the bond. The asset swap spread uses a *single* maturity swap rate to measure the credit risk. Although the Z-spread is better suited to risky discount factors, markets prefer to use the asset swaps as a measure of credit risk. The reason is that the markets do *not* quote the credit spread as a spread to *zero-coupon* swap rates. The credit spread is quoted as a *spread to a par swap rate* because the par swaps are much more liquid than the zero-coupon swaps.

4. Real-World Complications

Credit markets and credit derivatives trading are inherently more complicated and heterogeneous than most other markets, and one faces an unusual number of real-life complications that

[12] Although the risky discount factors are calculated differently from the risky DV01 we will see in this chapter.

theoretical models may not account for. In this section we look at some of the real-life aspects of CDS contracts.

Contractual equation (6) provides a natural hedge for the CDS and shows one way of pricing it. Similar contractual equations may provide usable hedges and pricing methods for some bread-and-butter instruments with negligible credit risk, but for CDSs these equations are essentially *theoretical*. The simplified approach discussed above may sometimes misprice the CDS and the hedge obtained earlier may not hold. There may be several reasons why the *benchmark spreads*[13] on this credit may deviate significantly from the CDS rates. We briefly discuss some of these reasons.

1. In the preceding example, the CDS had a maturity of 3 years. What if the particular credit had no outstanding 3-year bonds at the time the CDS was issued? Then the pricing would be more complicated and the benchmark spread could very well deviate from the CDS rate.
2. Even if similar maturity bonds exist, these may not be very liquid, especially during times of high market volatility. Then, it would be natural to see discrepancies between the CDS rates and the benchmark spreads.
3. The tax treatment of corporate bonds and CDSs are different, and this introduces a wedge between the corresponding spread and the CDS rate.
4. As mentioned earlier, CDSs result in physical delivery in the case of default. But this delivery is from a basket of deliverable bonds. This means that the CDS contains a *delivery option*, which was not built into the contractual equation presented earlier.

In reality, another important issue arises. The construction of the synthetic shown above used a money market account that was assumed to be risk-free. In general, such money market accounts are almost never risk-free and the deposit-accepting institution will have a default risk. This introduces another wedge between the theoretical construction and actual pricing. This additional credit risk that creeps into the construction can, in principle, be eliminated by buying a new CDS for the deposit-accepting institution.

4.1. Restructuring

Another real-life complication deals with the definition of default itself. Credit events are normally *failure to pay* and *bankruptcy*. Any nonpayments of interest or principal would count as the former and any type of formal bankruptcy would count as the latter. In our theoretical engineering we used this second definition of defaults. Yet, in reality, most single-name CDSs also consider restructuring as an additional default event. Further complicating the picture is the type restructuring, summarized below.

There are three types of restructuring clauses in the CDS contracts. The first is simply no-restructuring, *No R*. In this case any structuring would not constitute a credit event. Normally, high yield CDS typically trade No R. This is especially true of the CDX indexes in the United States.

The second type is modified restructuring, *Mod R*. This creates new conditions for a credit event.[14]

The third is the modified modified restructuring, *Mod Mod R*. In this case the maturity limits on the deliverable bonds or loans are somewhat different. There is an exception that the

[13] Over the swaps.

[14] Here there is a condition that on a restructuring event, the maximum maturity of the delivered bonds is no more than 30 months after the event, although there are exceptions.

bonds (loans) with a maturity of more than 30 months but no more than 60 months can be delivered.

Obviously CDS contracts with restructuring will have higher CDS spreads than the contracts of the same name, without restructuring. Note the key difference: in a bankruptcy or failure-to-pay type credit event the price differences of deliverable bonds will be relatively small. In a restructuring, the deliverable bonds could have very different values depending on the maturity. The protection buyer has the option to deliver the cheapest bond and hence this option could be very expensive.

4.2. *A Note on the Arbitrage Equality*

The simple contractual equation derived earlier suggests that we should have

$$Co_{t_0} - s_{t_0} = c_{t_0} \tag{17}$$

under ideal conditions.

Yet, in reality, even when bonds are selling close to par, we in general observe

$$Co_{t_0} - s_{t_0} < c_{t_0} \tag{18}$$

Under these conditions instead of buying credit protection on the issuer, the client would simply short the bond and get in a receiver swap. This will provide the same protection against default, and, at the same time, cost less. So why would clients buy COSs instead? In fact, such inequalities can be caused by many different factors, briefly listed below.

1. CDS protection is "easy" to buy. On the other hand, it is "costly" to short bonds. One has to first go to the repo market to find such bonds, and repo has the mark-to-market property. With CDS protection, there is no such inconvenience.
2. Shorting a bond is risky because of the possibility of a short squeeze. If too many players are short the bond, the position may have to be covered at a much higher price.
3. Some bonds may be very hard to find when a sudden need for protection arises.
4. Also, as discussed earlier, a delivery option premium is included in the CDS rate.

These factors may cause the theoretical hedge to be different from the CDS sold to clients. Finally, it should be noted that when the probability of default becomes significant CDS dealers may suddenly move their prices out and stop trading. In more precise terms, the market moves from trading default toward trading the *recovery*. This is done by quoting the implied *upfronts* (UPF) instead of spreads.

5. CDS Analytics

We will discuss the main quantitative tools used in CDS pricing and hedging using a 3-year, single-name CDS. The idea is to illustrate the way (risk-adjusted) *probability of default* is obtained and used. Also, we would like to determine the so-called *risky DV01* and *risky annuity* factors. These factors are used in obtaining hedge ratios and during the CDS pricing.

Let c_t be rate of a single-name CDS at time t. Let R denote the fixed recovery rate, and N be the notional amount. As usual $B(t_0, t_i)$ with $t_0 < t_i$ represents the default-risk free pure discount bond prices at time t_0. The bonds mature at times t_i and have par value of \$1. First we develop the notion of the default probability p_t at time t.

6. Default Probability Arithmetic

Modeling the occurrence and the timing of default events can be quite complex. The market, however, gravitates to some simple and tradeable notions as we have seen before with options and implied volatility. In this section we discuss the arithmetic behind the trading of default probability.

Imagine default as an event that happens at a random time τ, starting from some time t_0. How should we model the probabilities associated with such events?

We consider the *exponential distribution*, which can be described as a probability distribution describing the *waiting time* between events. In this case the event is the default and the waiting time is time until default. Thus we are dealing with modeling the random times until some events occur.

Let the τ be the occurrence time of a default. The density function of an *exponentially distributed* random variable, $f(\tau)$, is given by

$$d(\tau) = \lambda e^{-\lambda\tau} \tag{19}$$

This is one way to model default occurrence and timing.

We see that as the parameter λ gets bigger, the probability that the event will occur earlier goes up. Hence this parameter governs when the default event is likely to occur. It is called the *intensity* associated with the random event.

The market does not like to use the exponential distribution. Taking the second order Taylor series approximation of the $f(\tau)$ around the point $\tau_{t_0} = 0$ we get

$$f(\tau) \cong \lambda - \lambda^2\tau + \frac{\lambda^2\tau^2}{2} + o(\tau)^3 \tag{20}$$

Thus the probability that default will occur in a small interval Δ immediately following τ_{t_0} is approximately given by

$$f(\tau_{t_0})\Delta \cong \lambda\Delta \tag{21}$$

Integrating the density $f(\tau)$ from 0 to some T we get the corresponding probability distribution function (PDF)

$$\begin{aligned} P(\tau < T) &= \int_0^T \lambda e^{-\lambda\tau} d\tau \\ &= 1 - e^{-\lambda T} \end{aligned} \tag{22}$$

The first order Taylor series approximation of the PDF is then given by

$$1 - e^{\lambda\tau} \cong \lambda\tau \tag{23}$$

According to this, the probability that default occurs during a period Δ is approximately proportional to λ. The probability that the event will occur within one year is obtained by replacing the T in the PDF by 1. We obtain

$$P(\tau \leq 1) \cong \lambda \tag{24}$$

Hence the parameter λ can be looked at as the approximate constant rate of default probability. The market likes to trade *annual* default probabilities, assuming that the corresponding probability is constant over various trading maturities. Obviously as time passes and quotes change, the corresponding default probability will also change. Hence it is best to use the subscript t_0 to denote a probability that is written in an instrument at time t_0, and use $p_{t_0} \cong \lambda_{t_0}$ as the default probability written in a contract at inception time t_0.

Looking at this from a different way: Suppose $0 < \Delta$ is a small time interval. The default event is represented by a random variable d_t that assumes the values of zero or one, depending on whether during $[t, t+\Delta]$ the credit defaults or not.

$$d_t = \begin{cases} 0 & \text{No default} \\ 1 & \text{Default} \end{cases} \tag{25}$$

Now we make the assumption that the probability of default follows the equation

$$\text{Prob}(d_t = 1) \cong p_t \Delta \tag{26}$$

which says that the probability that the credit defaults during a small interval Δ depends on the length of the interval and on a parameter p_t called the "intensity." According to this the probability that default occurs by time t will be given by

$$\text{Prob}(\tau < t) = 1 - e^{-\int_0^t p_s ds} \tag{27}$$

Assuming p_s is constant gives

$$\text{Prob}(\tau < \Delta) = 1 - e^{-p\Delta} \tag{28}$$

We have the Taylor series approximation around $t = 0$

$$e^{-p\Delta} = 1 - p\Delta + \frac{1}{2}p^2\Delta^2 \ldots \tag{29}$$

or

$$1 - e^{-p\Delta} \cong p\Delta \tag{30}$$

According to this the probability that the credit defaults during one year will equal p.

6.1. The DV01's

Working with a CDS of maturity $T = 3$ years, let p_{t_0} be the risk-adjusted default probability associated with a CDS contract of maturity 3 years. We will ignore the t_0 subscript and write this parameter simply as p. The CDS rate is c_{t_0} and is observed in the markets. The δ_i is the time as a proportion of the year between two consecutive settlement dates.

Using this we can write the *initial* value of the CDS at inception time t_0 as follows. First, if during the three years the name does not default, the present value of the cash flows denoted by PV_{ND} will be

$$PV_{ND} = [B(t_0, t_1)(1-p)\delta_1 c_{t_0} + B(t_0, t_2)(1-p)^2\delta_2 c_{t_0} + B(t_0, t_2)(1-p)^3\delta_3 c_{t_0}]N \tag{31}$$

On the other hand, the name can default during years one, two, or three. The expected present value of the accrued premium denoted by PV_{AP} if a case default event occurs will be

$$\begin{aligned} PV_{AP} = [&B(t_0, t_0 + \Delta_1)\Delta_1 p c_{t_0} + B(t_0, t_1 + \Delta_2)\Delta_2(1-p)p c_{t_0} \\ &+ B(t_0, t_2 + \Delta_3)\Delta_3(1-p)^2 p c_{t_0}]N \end{aligned} \tag{32}$$

Three comments might help here. First, the Δ_i are the parameters that determine the pro-rated spreads that will be received. If the name defaults right after the settlement date then that particular Δ_i will be close to zero. If the default is right before the next settlement date, the Δ_i will be close to the δ_i.[15] Assuming that the expected default time is the middle of the settlement period gives

$$PV_{AP} = \frac{1}{2}[B(t_0, t_0 + \Delta_1)\delta_1 p c_{t_0} + B(t_0, t_1 + \Delta_2)\delta_2(1-p)pc_{t_0} + B(t_0, t_2 + \Delta_3)\delta_3(1-p)^2 pc_{t_0}]N \tag{33}$$

The $\frac{1}{2}$ comes from the expected default time during an interval $[t_i, t_{i+1}]$ as given by a uniform distribution on the interval [0,1], the height of the uniform density being dt.

The expected value of the compensation for the cash payouts during default will be given by

$$PV_D = B(t_0, t_0\Delta_1)p(1-R)N + B(t_0, t_1 + \Delta_2)(1-p)p(1-R)N + B(t_0, t_2 + \Delta_3)(1-p)^2(1-R)N \tag{34}$$

The expected payments and receipts should be equal under the *risk-adjusted* probability and the CDS rate c_{t_0} must satisfy the equation

$$PV_{ND} = PV_D - PV_{AP} \tag{35}$$

Generalizing from the three-year maturity to n settlement dates, we can write

$$\left[\sum_{i=1}^{n} B(t_0, t_i)(1-p)^i \delta_i c_{t_0}\right] N + \left[\frac{1}{2}\sum_{i=1}^{n} B(t_0, t_{i-1} + \frac{1}{2}\delta_i)(1-p)^{i-1} P\delta_i c_{t_0}\right] N = \sum_{i=1}^{n} B(t_0, t_{i-1} + \frac{1}{2}\delta_i)(1-p)^{i-1} p(1-R)N \tag{36}$$

Using equation (36) we now define two important concepts. The first is the *risky annuity* factor. Suppose the investor receives \$1 at all t_i. The payments will stop with a default. For that period the investor receives only a prorated premium. The risky annuity factor, denoted by $\tilde{A}$, is the value of this defaultable annuity. It is obtained by letting $c_{t_0}N = 1$ on the right-hand side of the expression in equation (36)

$$\tilde{A} = \left[\sum_{i=1}^{n} B(t_0, t_i)(1-p)^i \delta_i\right] + \left[\frac{1}{2}\sum_{i=1}^{n} B(t_0, t_{i-1} + \frac{1}{2}\delta_i)(1-p)^{i-1} P\delta_i\right] \tag{37}$$

The second concept is the *risky* DV01. This is given by letting the c_{t_0} change by one basis point.[16] Often the traders use the $\tilde{A}$ as the risky DV01. Yet, there is a difference between the two concepts. The risky *DV01* is how much the value of the CDS changes if one increases c_{t_0} by .0001. In general this is not going to equal $\tilde{A}$, although will be close to it depending on the shape of the curve and the change in spreads. The reason is the relationship between the probability of default p_{t_0} derived in the first chapter and the CDS spread c_{t_0},[17]

$$(1-R)p_{t_0} = c_{t_0} \tag{38}$$

[15] For example, $\frac{1}{4}$ if the CDS settles quarterly.

[16] The DV01 is quoted as if one basis point change is given by $d_{c_{t_0}} = 1$. In reality the basis point change would be .0001, but the market quotes it after multiplying by 10,000. As usual the decimal point is disliked by the trader during the trading process and eliminated from the quotes altogether.

[17] This assumes that defaults occur at the settlement dates only. Otherwise it is only an approximation.

If DV01 is the value of a stream of payments to a $dc_{t_0} = 1$, then note that we have

$$dp = \frac{1}{(1 - R)} \tag{39}$$

In other words, the risky annuity factor $\tilde{A}$ will change due to *two* factors. Both the c_{t_0} and the p_{t_0} would change. This means that the risky DV01 is a nonlinear function of the c_{t_0} and that there will be a convexity effect. Most market participants ignore this effect and consider the annuity factor $\tilde{A}$ as a good approximation of DV01. Yet, if the CDS spreads are moving in a volatile environment then the two sensitivity measures would differ.

6.2. *Unwinding a CDS*

There are three essential ways to unwind a CDS transaction. The first two are similar to the transactions we routinely see in futures markets. The third is a bit different.

The most common way of unwinding a CDS position is to offset the position with another CDS or by getting in an offsetting position in the underlying assets.

A second way to unwind a CDS position is by terminating the contract and pay (or receive from) the counterparty the present value of the remaining CDS cash flows. The trick here is to remember that in a credit event, the cash flows would terminate early, hence, the calculation of the PV should take this into account. This is a good example for the use of risky annuities and risky DV01's.

Finally, assigning the contract to another dealer is the third way of terminating the contract. Below we discuss an example for the use of DV01 by terminating the contract before maturity. We will calculate the upfront cash payment or receipt.

EXAMPLE:

Theoretically, one can unwind a CDS position by getting into an offsetting position or by receiving or paying the PV of the contract. However, in practice, if the CDS in question is substantially in the money, then it may be difficult to find a counterparty who will be willing to pay a substantial upfront for a position that can be attained with no upfront cash. This means that the original owner of the contract may have to accept a PV significantly lower to entice the counterparty to take over the position. In other words, there will be a haircut issue. For example:

An investor bought 3-year ABC protection on October 29, 2007 at 122 bps on \$100 notional. Three days later on November 1, the spread is at 140 bps. This means that the original CDS will be in-the-money by an amount.

If there is no credit event this means an annuity of

$$\frac{(140 - 122)}{10{,}000} 100\, m = \$180{,}000 \tag{40}$$

to be paid annually at times t_1, t_2, t_3. On the other hand, there may be a credit event and the coupon payments may stop. Assuming that this credit event can occur only at the end of the year there are three possible cash flow paths.

$$\{180, 180, 180\}, \{180, 180\}, \{180\} \tag{41}$$

The payments during a default event will approximately cancel each other out assuming that the same cheapest-to-deliver bond is involved during the delivery.

Suppose the recovery rate is 40%. In order to determine the present value (PV) of these cash flows we use risk-adjusted probabilities of default p obtained from the CDS spread at time t_0

$$p = \frac{142}{10{,}000(1-.4)} = .023 \tag{42}$$

Clearly, we also need the corresponding risky zero bond prices, $\tilde{B}(t_0, t_i)$. *Suppose they are given by*

$$\tilde{B}(t_0, t_1) = .92 \tag{43}$$

$$\tilde{B}(t_0, t_2) = .86 \tag{44}$$

$$\tilde{B}(t_0, t_3) = .79 \tag{45}$$

Hence we can write the present value of the coupon payments if no default occurs,

$$\begin{aligned} DV01 = {} & ((\tilde{B}(t_0, t_1) + \tilde{B}(t_0, t_2) + \tilde{B}(t_0, t_3))(1-p)^3 \\ & + (\tilde{B}(t_0, t_1) + \tilde{B}(t_0, t_2))(1-p)^2 + (\tilde{B}(t_0, t_1))(1-p)).018(100) \end{aligned} \tag{46}$$

where the $p^i, i = 1, 2, 3$ *are the probabilities associated with each annuity path. Plugging in the numbers we obtain*

$$\mathrm{DV01} = 89.9 \tag{47}$$

Note that we needed to use the default risky discounts and the corresponding DV01 because the default will change the timing of the cash flows.

6.3. *Credit Curve Strategies*

In this section we consider two standard fixed income strategies applied in the credit sector. The first deals with changes in the *slope* of the credit curve, while the second deals with changes in the *curvature* of the curve.

Just as in the case of standard fixed income products, the credit curve allows for curve flattener and curve steepener trades. The idea is self explanatory and follows the fixed income swap positions applied to the iTraxx curve.

EXAMPLE:

Consider Hutchison Whampoa credit-curve flattener via bond-basis play, exploiting value in long end while protecting against any general market deterioration. Buy Hutch 2027 bonds (cheapest on curve), buy 5-year CDS protection. Spread now 82 bps (asset-swap bond spread 131 bps versus CDS at 49 bps). Target compression to 65 bps in coming months. Exit if spread widens to 90 bps.

Trade primarily a play on long-end bonds: buying protection at shorter end offers hedge in case of bad news on Hutch or general bond downturn. CDS cheaper alternative to selling short-end bond given lack of repo liquidity; using asset swap avoids fixed-rate risk at short end. Spread should slowly grind tighter as more investors switch out of shorter Hutch bonds (2010, 2011, 2014 maturities) where value has already been squeezed out into undervalued longer end.

The example below is a strategy that deals with changes in the curvature of the credit curve. It uses a barbell or butterfly position to do this.

EXAMPLE:

If the synthetic CDO pipeline picks up and focuses on the 7-year maturity, the 5s-7s segment of iTraxx could be too steep and the 7s-10s too flat. If so, the bank says investors could consider an iTraxx butterfly trade.

Buy iTraxx Europe 5-year protection (default exposure EUR28.433027) and 10-year protection (EUR21.566973) at 36 bp and 58.5 bps respectively, and sell 7-year protection at 46.75 bp for a duration and default neutral trade with positive carry and slide.

7. Structured Credit Products

CDS is the basic building block of the credit sector. Using CDS engineering one can immediately create credit-market equivalents of risk-free fixed income instruments. Some of these instruments are discussed in this section.

7.1. Digital CDS

Credit default swaps with fixed recovery rates are called *Digital CDS*. These products have several advantages. They have lower costs, more precise focus on the credit risk, and greater transparency. They will not be subject to difficulties associated with the auctioning process after a default event.

Essentially the digital CDS eliminate the random recovery rates associated with the conventional CDS. In fact, recent events suggest the possibility that the recovery rates associated with CDS and synthetic CDOs have higher volatility than do recovery rates implied by corporate bond defaults. Digital CDS would be immune to such basis risk.

7.2. Credit Options

It is natural for options to be the first derivative to be written on credit indices. After all, there are liquid indices and these could serve as an attractive underlying for those who would like to hedge their credit volatility or for investors who would like to take positions in them. Yet, such options turn out to be much more complicated to structure and market than visualized at the outset. Although there is decent liquidity in the market, with daily references to iTraxx and CDX implied volatility, some difficulties remain.

There are essentially three problems associated with options on credit indices. First, the credit sector is heavily influenced by monetary policy and has a long credit cycle. The practitioners would need long-dated options. Second, although stocks are liquid, a large majority of corporate bonds have very little liquidity; but the third and main problem is the *index roll*. Essentially these indices change every six months and one cannot price a long-term option against such an unstable benchmark.

The credit option trader is then forced to operate in the shortest maturities and the exercise dates controlled by the roll dates.[18] This is not sufficient for traders since their needs

[18] Two months before the next roll, the expirations can be at most two months.

are really five-year options because these could be used to arbitrage the structured credit market.[19]

There is another important difference between credit index options and options from other markets. In the credit sector the longer-dated an option, the more it becomes a correlation product. With longer-dated options the underlying risk is to what extent referenced credits will move jointly. Shorter-dated options, on the other hand are more like volatility products.

7.3. *Forward Start CDOs*

Using forward start CDOs one can take leveraged positions on the outlook for credit spreads in the distant future, say from 2010 to 2015. Such forward start products may be useful for some investors that want to hedge their positions on take exposure during the *credit cycle*.

The product can be structured by selling credit default swaps maturing, say, in ten years, and buying CDSs maturing in a shorter maturity, say, five years. Such forward start instruments are marketed as bespoken deals on the iTraxx or CDX default swap indexes. The most common reference is the mezzanine tranche insurance against the 3–6% of defaults in a credit portfolio.

The net position of buying five-year protection and selling ten-year protection is selling five-year protection five years from now. Note that such a position will have positive carry. Due to this, such trades become popular if the iTraxx curve is relatively steep.

7.4. *The CMDS*

The constant maturity default swap (CMDS) is an important component of the structured credit sector. A CMDS can be structured as follows.

Fix the maturity of the CDS, say, 5 years. Consider a series of T-maturity CDSs starting at times $t, t+1, t+2, t+3, t+4$ and $t+5$. Note that the spread of the current CDS is known at time t, whereas future CDS spreads c_{t+i} will be known only in the future as time passes. Also note that these CDSs all have the same five-year constant maturity. Let their spread be denoted by c_{t+i}. Then the CMDS will be the five-year CDS that pays the floating spreads c_{t+i}.

Essentially this is an extension of the CMS swaps to the case where the underlying risk incorporates default risk. There are several uses of this crucial component. A market example follows.

EXAMPLE:

Investors want to sell protection today with the potential to take advantage of expected future spread-widening; this is exactly what the CMDS product provides. Client interest in constant maturity CDOs (CM-CDOs) also helps establish the CMDS market by drawing clients' attention to what the product can achieve.

Nearly all new CDOs in the pipeline today come with the option of constant maturity technology embedded in them.

[19] One proxy for long dated options is the range accrual that has been marketed to players who want to arbitrage the structured credit sector. These notes pay a coupon, or *accrue*, depending on the number of days the index I_t remains within a prespecified range $[L^{\min}, L^{\max}]$. Note the roll problem can be solved here by resetting the range $[L^{\min}, L^{\max}]$ at every roll.

Concerns about P&L volatility that may be injected into synthetic CDOs because of IAS 39 drives a lot of the interest in CM-CDOs because IAS 39 requires all derivatives to be marked-to-market through the income statement. Using CMDS to mitigate mark-to-market volatility is a legitimate reason for employing them. Market participants do not expect CMDS to outshine the market for tranched credit index products or credit options. But having standardized fixings would be useful both for closing CDO deals and for developing the market for cash-settled credit spread options and other structured credit and volatility products.

CMDS are generally viewed as a building block for other structured credit products. The challenge is that fixings for credit derivatives are not as straightforward as they are for interest-rate derivatives. "The amount of information that you need is so much greater than it is in rates. And where do you stop with fixings? The universe of names in the credit default swap market is very large. How many do you fix, and do you fix for the five-year or the whole curve for each name?" asked a trader.

Fixings on the credit indices are relevant for CMDS contracts because if CDSs are written on indices, which most are, an independent fix is needed. Quoting an index, such as Dow Jones iTraxx, is not easier as the coupon must be linked to 125 names, but the liquidity in credit indices is greater than it is in the underlying single names.

Standardized fixings. *About 18 dealers have been working with Credited, the electronic dealer-broker, and Mark-It Partners, the OCT derivatives valuation firm, to standardize fixings for credit indices. Six weekly fixing test runs have been completed so far (IFR 1552). No runs on single name credit default swaps have been tested yet, though.*

According to dealers, the standard resets that are being developed are essential for the continued growth of the product as opposed to relying on just dealer polls as documented in current contracts.

"Fixings are worthwhile for the credit derivative market not just for CMDS but for all other second and third generation credit derivative products. For example, if fixings are done for iTraxx index tranches, this will help breed a further range of derivatives on the tranches. So, in this way, fixings are essential just like Libor fixes every day for the swap market," one dealer added.

For the CMDS market to evolve, common documentation must also be forthcoming, dealers say. In CMDS, two documentation issues to overcome are whether the coupon is quarterly or semiannual and if the fixing is $T + 1$ *or* $T + 2$*.*

The CMDS can be used to take an exposure to the movements of the credit curve. If one expects the credit curve to steepen then one could, for example, buy a five-year protection on the CMDS that references a ten-year CDS and sell protection on the five-year CMDS that references a three-year CDS. The reverse could be done if the credit curve is expected to flatten. One could also put together a swap of the CDSs: for example, paying ten-year, and receiving three-year reference spreads.

7.5. *Leveraged Super Senior Notes*

The spreads on super senior tranches are very tight—say, 7–9 bps. This is not very attractive to the investors. Hence, the demand for super senior tranches is relatively low. Yet, banks have issued many mezzanine tranches on CDOs and have kept the super senior and equity tranches on their books. They need a way to generate a demand for these tranches. Leveraged super senior notes is one method that was devised for selling the super senior risk to others.

With the note investors, take the additional exposure to the mark-to-market value of the tranche, and the trigger protects the bank against the investor's credit risk. Given the leverage, super senior investment may lose an amount greater than the original investment. With the trigger, this risk is reduced.

In this sense one can say that a leveraged super senior note is a modification of the super senior tranche. This modification occurs first in the leverage. N is collected from the investor, but λN with $1 < \lambda$ invested in the super senior tranche. Hence the return is also multiplied by λ. Second, there is a trigger on the market value of the note. If this trigger is reached the issuer of the note can unwind the position and return the mark-to-market value of the note to the investors.

EXAMPLE:

Consider a USD1 billion portfolio. Let the senior tranche have attachment points of 12% and 30%. This gives a thickness of USD180 million.

Suppose we select a leverage ratio of 10. The leveraged super senior note will consist of an issue of USD18 million. This amount is multiplied by 10 and invested as a notional amount of 180 million in the super senior tranche. Assuming that the quoted spread for this tranche is 8 bp, the note will pay Libor + 100.

The market value trigger could be defined, for instance, as 70% of the issue amount of USD18 million. If the market value of the note falls below this limit, say becomes 12, then the 12 is returned to the investors instead of the original investment of 18.

There are two general tendencies in structured credit. The first is to introduce *leveraged transactions*; the second is the introduction of *market risk* in addition to default risk. The leveraged super senior notes are examples of the first tendency. The CPPI techniques applied to credit is an example of the second and they will be discussed in a later chapter.

Where does risk lie in leveraged super senior notes? Owing to the substantial credit protection inherent in a super senior structure, the default risk itself is very limited. The risk borne by investors mainly lies in the behavior of the *market value* of the CDS tranche, which depends on spreads and correlations. Most transactions are actually structured so as to ignore correlation variations as market value parameters; they introduce instead a *trigger* on the portfolio average spread.

In such a trade, the investor is long the super senior risk, while the dealer goes short that risk. To boost the return on these investments, dealers have been constructing products for their clients using borrowed funds.

During the year 2000 dealers purchased significant amounts of mezzanine protection and, as a result, were left exposed to the senior and equity components of the capital structure. This is because they had marketed mezzanine products to clients and kept the senior and equity tranches on their books. Note that leveraged super senior trades is one way of *buying* protection on the senior and super senior tranches. Hence, the structured product is in fact useful to both the client and the dealer.

If dealers did not have such long senior and super senior positions, after buying protection from a client with a leveraged super senior issuance, they would hedge themselves through the iTraxx index or other individual credit default swaps, although there may be a significant basis risk between the two risks.

7.6. EDS versus CDS

Equity default swaps (EDS) have been marketed by dealers with mixed success thus far. The EDS emulate credit default swaps. An EDS is "exercised" when a company's stock price S_t falls below a prespecified barrier H. Normally this barrier will be 30–40% below the current stock price level. If $S_t < H$ happens, then an "equity event" similar to a credit event takes place. Note, however, that hitting an equity barrier H, no matter how distant it is, is more likely than a credit event. After all, a company's stock price can fall dramatically without the company going bankrupt.

Again, similar to CDSs, dealers can, and have tried to, put together CDOs of the equity default swaps. Such CDOs can get rated. Normally, however, it is difficult for dealers to get a big enough tranche of such a CDO rated higher than A.[20]

It is worth adding that the EDS structure is very similar to a deep out-of-the-money put option written on the stock. In both cases there is a barrier, namely the strike price in the case of the option, such that the option buyer receives a cash payment. The major difference is perhaps the expiration date of the EDS which can be much longer.

8. Total Return Swaps

Total return swaps (TRS) trade *default*, *credit deterioration*, and *market risk* simultaneously. It is instructive to compare them with CDSs. In the case of a CDS, a protection buyer owns a bond issued by a credit and would like to buy insurance against *default*. This is done by making constant periodic payments during the maturity of the contract to the protection seller. It is similar to, say, fire insurance. A constant amount is paid, and if during the life of the contract the bond issuer defaults, the protection seller compensates the protection buyer for the loss and the contract ends. The compensation is done by paying the protection buyer the face value of, say, 100, and then, in return, accepting the delivery of a deliverable bond issued by the credit. In brief, CDSs are instruments for trading defaults only.[21]

A total return swap has a different structure. Consider a bond or any arbitrary risky security issued by a credit. This security makes two types of payments. First, it pays a coupon interest. Second, there will be associated capital gains (appreciation in asset price) and capital losses (depreciation in asset price), which include default in the extreme case. In a TRS the *protection seller* pays any depreciation in the asset price during periodic intervals to the protection buyer. Default is included in these payments, but it is not the only component. In general, assets gain or lose value for many reasons, and this does not mean the issuer has defaulted or will default. Nevertheless, the protection seller will compensate the protection buyer for these losses as well.

However, in a TRS, the protection seller's payments will not stop there. The protection seller will also make an additional payment linked to Libor plus a spread.

The *protection buyer*, on the other hand, will make periodic payments associated with the appreciation and the coupon of the underlying asset. Normally, asset prices appreciate and pay coupons more often than decline, but this is compensated by the Libor plus any spread received.

8.1. *Equivalence to Funded Positions*

The TRS structure is equivalent to the following operation. A market participant buys an asset, S_t, and funds this purchase with a Libor-based loan. The loan carries an interest rate, L_{t_i} and has to be rolled over at each t_i. The market participant is rated A– and has to pay the credit spread d_{t_0} known at time t_0. The S_t has periodic (coupon) payouts equal to c. The market participant's

[20] Rating agencies normally stress test instruments by putting them through scenarios. There are three basic scenarios: *normal*, *stress*, and *crash*. It is very difficult for a CDO of EDS to stay highly rated during a stock market crash scenario.

[21] Maturity is typically five years in the case of most corporate credits.

net receipts at time t_{i+1} would, then, be the following:

$$(\Delta S_{t_{i+1}} + c) - (L_{t_i} + d_{t_0}) S_{t_0} \Delta \tag{48}$$

where the $\Delta S_{t_{i+1}}$ is the appreciation or depreciation of the asset price during the period, $\Delta = [t_i, t_{i+1}]$. The c is paid during Δ. The payments are in-arrears.

A TRS swap is equivalent to this purchase of a risky asset with Libor funding. Except, in this particular case, instead of going ahead with this transaction, the market participant can simply sign a TRS with a proper counterparty. This will make him or her a protection seller. Banks may prefer these types of TRS contracts to lending to market practitioners.

The following is a specific example from the CDS market. Argentina, WorldCom, and Enron are all interesting names to be associated with the CDS market because of the large size of the respective defaults. This example deals with Argentina, where the CDS rate was around 40% for one year around the default period.

EXAMPLE:

One-year Argentina credit default swap mid-levels hit 4,000 bp late last week, though the highest trade in the sovereign is thought to have been a one-year deal at 2,350 bp early in the week.

Derivatives market-makers were cautiously quoting default swap prices on an extremely wide bid/offer spread (the two-year Argentina mid rose to around 3,900 bp), but mostly concentrated on balancing cash market hedges, which did not prove easy.

Dealers who have sold protection also consulted their lawyers to plot tactics in the event that Argentina defaults, or restructures its debt. It is likely that more than US$1bn of credit default protection on Argentina has traded in the last few years, which could result in the biggest default swap payout yet, if there is a clear-cut default or debt restructuring. There is plenty of scope for disagreement on whether or not the payout terms of swaps have been met, however, depending on how any debt restructuring is handled by the Argentine authorities.

Pricing default swaps when a payout trigger could be hours away is an art, not a science. Late last week traders were working from the closing price on Thursday of Argentina's FRBs of 63.5, which was the equivalent of 3,060 bp over Libor, then adding a 30–40% basis for the theoretical risk of writing a default swap, as opposed to the asset swap value of a bond trade. For much of this year, traders have been using a default against asset swap basis of around 10% of the total spread for deals in Latin American sovereigns. (IFR, July 2001)

9. Conclusions

This chapter is only a very brief introduction to this important class of credit derivatives. We saw that credit default swaps play a key role in completing the methodology of financial engineering. At the same time, we discussed the role of total return swaps as funding tools.

Suggested Reading

Several recent books deal with this new sector. For a good theoretical background and some empirical work, ***Duffie and Singleton*** *(2003) is very useful.* ***Bielecki and Rutkowski*** *(2001) is* *more mathematically involved, but excellent. For an introduction to some of the market terminology and new products, the reader can consult* ***Tavakoli*** *(2001) or* ***Das*** *(2000). The monthly Risk publication, Credit, is also good reading on market activity. In this chapter we did not cover pricing. The classic reading here is* ***Merton*** *(1974).* ***Giesecke*** *(2002) is a good survey on pricing. The reader should also consult the very good source,* ***Schonbucher*** *(2003).*

Exercises

1. This exercise deals with value-at-risk calculations for credit portfolios. Using the data on a corporate financial statement, answer the following questions:

   (a) How would you calculate the default probabilities?
   (b) How can one obtain the migration matrix for a credit?
   (c) How can one obtain the joint migration probabilities for the relevant credits in a bank's portfolio?

2. You are given two risky bonds with the following specifications:

   Bond A
   (a) Par: 100
   (b) Currency: USD
   (c) Coupon: 10
   (d) Maturity: 4 years
   (e) Callable after 3 years
   (f) Credit: AA–

   Bond B
   (a) Par: 100
   (b) Currency: DEM
   (c) Coupon: Libor + 78 bp
   (d) Maturity: 5 years
   (e) Credit: AAA

   You will be asked to transform Bond A into Bond B by acquiring some proper derivative contracts. Use cash flow diagrams and be precise.

   - Show how you would use a currency swap to switch into the right currency.
   - Show how you would use an interest rate swap to switch to the needed interest rate.
   - Is there a need for using a swaption contract? Can the same be accomplished using forward caps and floors?
   - Finally, show *two* ways of using credit derivatives to switch to the desired credit quality.

3. Consider the following reading, which deals with collateralized debt obligations (CDOs).

   > *Despite the deluge of downgrades in the collateralized debt obligation (CDO) market, banks are not focusing on the effect of interest rate swaps on arbitrage cash flow CDOs, Fitch Ratings said in a report released last week.*
   >
   > *Ineffective interest rate hedging strategies inflicted the hardest blows to the performance of high-yield bond CDOs completed during 1997–1999, the report noted.*
   >
   > *This combination of events caused some CDOs to become significantly over-hedged and out-of-the-money on their swap positions at the same time, the report found. For its report, Fitch used a random sample of 18 cash-flow deals that recently experienced downgrades.*

While half the CDOs benefited from falling rates, half did not. All nine of the over-hedged CDOs were high-yield bond transactions that closed before 1999.

With the benefit of hindsight, a balanced guaranteed or customized swap would have mitigated the over-hedged CDO's risks. Plain vanilla swaps, which were economically advantageous during 1997–1999, ended up costing money in the long run because the notional balance of the swap is set at the deal's closing date and does not change over time, the report said. CDOs tend to use a plain vanilla swap instead of a customized swap because they are cheaper. (IFR Issue, 1433, May 2002)

(a) Show the cash flows generated by a simple CDO on a graph. Suppose you are *short* the CDO.
(b) What are your risks and how would you hedge them?
(c) Show the cash flows of the CDO together with a hedge obtained using a plain vanilla swap.
(d) As time passes, default rates increase and interest rates decline, what happens to the CDO and to the hedge?
(e) What does the reading refer to with buying a customized swap?

4. (a) Consider the following quote from Reuters:

The poor correlation between CDS and cash in Swedish utility Attentat (VTT.XE) is an anomaly and investors can benefit by setting up negative basis trades, says ING. 5-yr CDS for instance has tightened by approx. 5 bp since mid-May while the Attentat 2010 is actually approx. 1 bp wider over the same period.

Buy the 2010 bond and CDS protection at approx midas +27 bp. (MO)

i. Display this position on a graph with cash flows exactly marked.
ii. Explain the logic of this position.
iii. Explain the numbers involved. In particular, suppose you have 100 to invest in such a position, what would be the costs and expected returns?
iv. What other parameters may have caused such a discrepancy?

(b) Explain the logic behind the two following strategies using cash flow diagrams.

Sell DG Hyp 4.25% 2008s at 6.5 bp under swaps and buy Landesbank Baden-Wuerttemberg 3.5% 2009s at swaps-4.2 bp, HVB says.

The LBBW deal is grandfathered and will continue to enjoy state guarantees; HVB expects spreads to tighten further in the near future.

Also, WestLB mortgage Pfandbriefe trade too tight. Sell the WestLB 3% 2009s at 5.4 bp under swaps and buy the zero risk weighted Land Berlin 2.75% 2010s at 2.7 bp under. (TMA)

(c) The following quote deals with implied forward rates in the credit sector. Using proper diagrams explain what the trade is.

Implied forward CDS levels look high because shorter-dated CDS are currently too cheap to 5-year, says BNP Paribas. Using the iTraxx Main curve *as reference gives a theoretical 3-year forward curve that shows 6-month and 1-year CDS both at 60 bp.*

"In 3-years time, we find that 6-month and 1-year CDS are very unlikely to be trading above 60 bp."

Take advantage through the 3-5-10-year barbell, buying iTraxx 3-year at 20.75 bp for EUR20M, selling iTraxx 5-year at 38 bp for EUR25M, and buying iTraxx 10-year at 61.25 bp for EUR7M.

The trade has a yearly carry of EUR32,000 for a short nominal exposure of EUR2M.

CASE STUDY: Credit-Linked Notes

Overall, this case study deals with credit default swaps, synthetic corporate bonds, and, more interestingly, credit-linked notes.

The case study highlights two issues.

a. Cash flows and the risks associated with these instruments, and the reasons why these instruments are issued.

b. The arbitrage opportunity that was created as a result of some of the recent issuance activity in credit-linked notes.

Focus on these aspects when answering the questions that follow the readings.

Reading

Default swap quotes in key corporates have collapsed, as a rush to offset huge synthetic credit-linked notes has coincided with a shortage of bonds in the secondary market, and with a change in sentiment about the global credit outlook. The scramble to cover short derivatives positions has resulted in windfall arbitrage opportunities for dealers who chanced to be flat, and for their favored customers.

At least E5bn, and possibly as much as E15bn, equivalent of credit-linked note issuance has been seen in the last month. The resulting offsetting of short-credit default swap positions has caused a sharp widening in the negative basis between default swaps and the asset swap value of the underlying debt in the secondary bond market.

Dealers with access to corporate bonds have been able to buy default swaps at levels as much as 20 bp under the asset swap value of the debt, and to create synthetic packages for their clients where, in effect, the only risk is to the counterparty on the swap. Credit derivatives dealers who chanced to be flat have been turning huge profits by proprietary dealing—and from sales of these packages to their favored insurance company customers.

Deutsche Bank, Merrill Lynch, Bear Stearns, and Citigroup have been among the most aggressive sellers of default swaps in recent weeks, according to dealers at rival houses, and their crossing of bid/offer spreads has driven the negative default-swap basis to bonds ever wider.

A E2.25bn credit-linked note issued by Deutsche Bank is typical of the deals that have been fuelling this movement. The deal, Deutsche Bank Repon 2001–2014, offered exposure to 150 separate corporate credits, 51% from the US and 49% from Europe. Because DB had the deal rated, the terms of the issue spread across trading desks in London and New York, and rival dealers pulled back their bids on default swaps in the relevant corporates.

Other banks were selling similar unrated (and therefore, private) credit-linked notes at the same time, which led to a scramble to offset swap positions. Faced with a shortage of bonds in the secondary market, and repo rates at 0% for some corporate issues, dealers were forced to hit whatever bid was available in the default swap market, pushing the negative basis for many investment grade five-year default swaps from an 8 bp–16 bp negative basis to a 12 bp–20 bp basis last week.

This produced wild diversity between default swaps for corporates that had seen their debt used for credit-linked notes, and similar companies that had not. Lufthansa five-year default swaps were offered at 29 bp late last week, while British Airways offers in the same maturity were no lower than 50 bp, for example.

Many default swaps were also very low on an absolute basis. Single A-rated French pharmaceuticals company Aventis was quoted at 16 bp/20 bp for a five-year default swap at the close of dealing on Friday, for example. Other corporate default swaps were also at extremely tight *levels, with Rolls-Royce offered as low as 27 bp in the five-year, Volkswagen at 26 bp, BMW offered at least as low as 26b, and Unilever at 21 bp.*

Run for the door

The movement was not limited to European credits. Offsetting of default swaps led to the sale of negative-basis packages in US names including Sears, Bank of America, and Philip Morris, with Bank of America trading at levels below 40 bp in the five-year, or less than half its trade point when fears about US bank credit quality were at their height earlier this year.

General market sentiment that the worst of the current downturn in credit quality has passed has amplified the effect of the default swap selling. Investors are happy to hold corporate bonds, which has left dealers struggling to buy paper to cover their positions as an alternative to selling default swaps.

"Everyone tried to run for the door at the same time," said one head dealer, describing trading in recent weeks. He predicted that the wide negative basis between swaps and bonds will be a trading feature for some time. Dealers worry that the banks which are selling default swaps most aggressively are doing so because they are lining up still more synthetic credit-linked notes. As long as they can maintain a margin between the notes and the level at which they can offset their exposure, they will keep hitting swap bids.

This collision of default swap-offset needs, a bond shortage, and improved credit sentiment is working in favor of corporate treasurers. WorldCom managed to sell the biggest deal yet from a US corporate last week, and saw spread talk on what proved to be a US$ 11.83bn equivalent deal tighten ahead of pricing. An issue of this size would normally prompt a sharp widening in default swaps on the relevant corporate, but WorldCom saw its five-year mid quotes fall from 150 bp two weeks ago to below 140 bp last week.

The decline in default swap quotes, and widening basis-to-asset swap levels for bonds, has been restricted to Europe and the US so far. If sentiment about the credit quality of Asian corporates improves there could be note issuance and spread movement. The dealers who have been struggling to cover their positions in the supposedly liquid US and European bond-and-swap markets may be reluctant to try the same approach in Asia, however.

With the prospect of more issuance of credit-linked notes on US and European corporates, and maintenance of the wide negative swap-to-bond basis, dealers who are allowed to run proprietary positions—and their insurance company clients—should reap further windfall arbitrage profits. The traders forced to offset deals issued by their structured note departments face further weeks of anxious hedging, however. (IFR, May 12, 2001)

Questions

(a) What is a credit-linked note (CLN)? Why would investors buy credit-linked notes instead of, say, corporate bonds? Analyze the risks and the cash flows generated by these two instruments to see in what sense CLNs are preferable.

(b) Suppose you issue a CLN. How would you hedge your position? Mention at least *two ways* of doing this. By the way, why do you need to hedge your position? Be specific.

(c) As a continuation of the previous question, why is whether or not the investors sell their corporate bonds important in this situation?

(d) Now we come to the arbitrage issue. What is the basis of the arbitrage argument mentioned in this reading? Be specific and explain in detail. Show your reasoning using cash flow diagrams.

(e) What does a 0% repo rate for some corporate paper mean? Why is the rate zero?

(f) Finally, why would this create an opportunity for corporate treasurers?